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by

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Using methods associated with Zernicke's polynomial expansion, we discussed atmospheric dispersion effects in dual wavelength adaptive optics systems. On the basis of dual frequency correlations associated with phase expansion coefficients, we solved for residual phase errors produced by atmospheric dispersion. The results clearly showed that, although atmospheric dispersion will cause phase compensation to be imperfect, the use, however, of light sources with wavelengths even shorter than the transmitting lasers as beacons, and, in conjunction with that, taking the product of the amount of beacon phase distortion and the specific value of the ratio λ_2/λ_1 (the beacon wavelength/ transmitted wavelength) to be the phase predistortion, it is possible to rectify relatively well the phase distortion of transmitted light beams.

Key Terms: Adaptive Optics, Atmospheric Dispersion, Residual Phase Error

I. FORWARD

In adaptive optics systems as well as in other infrared laser processes, one normally is made to use coaxial receiving and transmitting systems(1). In this type of system, the use of transmitted light and received light of the same wavelength will make surveying and control very difficult to realize. Because of this, one is often forced to use transmitted light and received light of different wavelengths. For example, in coherent adaptive optics technology, when correcting for atmospheric turbulence effects, one often uses phase distortion information for a type of wavelength (normally, it is visible light) to rectify the phase distortion of a different type of

wavelength (normally, it is infrared waveband). Due to the fact that atmospheric turbulence effects are related to wavelength, that is, existing atmospheric dispersion effects, rectification associated with this type of dual or double wavelength adaptive optics system will be imperfect. In order to discuss the influence of atmospheric dispersion effects on adaptive optics system characteristics, we, first of all, take the phase expansion in front of the wave and form a series of orthogonal Zernike polynomial sums. In conjunction with this, we discuss the correlation characteristics of expansion coefficients associated with lightwave phases for different wavelengths. From this, we carry out analyses and discussions on residual phase errors produced by atmospheric dispersion with the intention of supplying a theoretical foundation to choose appropriate laser wavelengths for the actual test manufacture of adaptive optics systems.

II. THE INTERRELATIONSHIPS OF DUAL FREQUENCY PHASE EXPANSION COEFFICIENTS

Assuming wave numbers which are respectively k_1 and k_2 for two beam light waves transmitted in a turbulent atmosphere a distance L after which the wave front phase distortions are respectively φ_1 and φ_2, the relationships between them can be expressed as[2]

$$\begin{aligned} B_\varphi(r, r', k_1, k_2) &= \langle \varphi_1(k_1, r)\cdot\varphi_2(k_2, r')\rangle \\ &= 4\pi^2 k_1 k_2 \int_0^L dz \int_0^\infty dk' k' \phi_n(k')\cdot J_0[k'(r-r')] \\ &\quad \cdot\cos\left[\frac{(L-z)k'^2}{2k_1}\right]\cdot\cos\left[\frac{(L-z)k'^2}{2k_2}\right]; \end{aligned} \tag{1}$$

In equation (1), $\langle\quad\rangle$ is an overall system average.

J_0 [ ] is a Type I zero order Bessel function.
$\phi_n(k')$ is the spectrum distribution associated with turbulent flows.. Assuming γ (unclear) = Rp, k'=k/R, ξ (unclear) = L $\cdot \eta$, R is the effective radius of the transmitted light beam. It is then possible to take equation (1), and, after turning it dimensionless, one obtains

$$B_\varphi(R\rho,\, R\rho',\, k_1,\, k_2) = \frac{4\pi^2 k_1 k_2 L}{R^2}\int_0^1 d\eta \int_0^\infty dk\cdot k\cdot \phi_n\left(\frac{k}{R}\right)\cdot J_0[k(\rho-\rho')] \cdot \cos\left[\frac{(1-\eta)Lk^2}{2k_1R^2}\right]\cdot\cos\left[\frac{(1-\eta)Lk^2}{2k_2R^2}\right], \tag{2}$$

Taking the phase distortion φ_j (j=1,2) expansions, one forms the Zernike polynomial form[3,4]

$$\varphi_j(\rho) = \sum_i a_i^{(j)}\cdot F_i(\rho/R); \tag{3}$$

In this, F_i is an i th order Zernike polynomial form. $a_{i(\text{unclear})}^{(1)}$ and $a_{i(\text{unclear})}^{(2)}$ are two expansion coefficients which correspond respectively to wavelengths λ_1 and $\lambda_2 (\lambda_j = 2\pi/k_j)$.
The correlation function for this type of dual frequency phase expansion coefficient is

$$\begin{aligned} C_{ii}(\lambda_1, \lambda_2) &\equiv \langle a_i^{(1)} a_i^{(2)}\rangle \\ &= \frac{4\pi^2 k_1 k_2 L}{R^2}\iint d^2K\, d^2K'\, Q_i^*(K)\, Q_i(K')\int_0^1 d\eta \int_0^\infty dk\, k\phi_n\left(\frac{k}{R}\right) \\ &\quad \cdot\cos\left[\frac{(1-\eta)Lk^2}{2k_1R^2}\right]\cos\left[\frac{(1-\eta)Lk^2}{2k_2R^2}\right] \\ &\quad \cdot\iint d^2\rho\, d^2\rho'\, \exp[2i\pi K\rho - 2i\pi K'\rho']\cdot J_0[k(\rho-\rho')]; \end{aligned} \tag{4}$$

In this, $Q_i(K')$ is a Fourier transform of Zernike polynomial forms. Q_i^* is Q_i's conjugate. Equation

(4) can be simplified to be

$$C_{ii}(\lambda_1, \lambda_2) = \frac{4\pi^2 k_1 k_2 L}{R^2} \iint d^2K' |Q_i(K')|^2 \int_0^1 d\eta \int_0^\infty dk k \phi_n\left(\frac{k}{R}\right) \cdot \cos\left[\frac{(1-\eta)Lk^2}{2k_1R^2}\right] \cos\left[\frac{(1-\eta)Lk^2}{2k_2R^2}\right] \cdot \frac{1}{R} \cdot \delta\left(K' - \frac{k}{2\pi}\right); \tag{5}$$

In equation (5), $\delta(\ )$ is function δ :

$$|Q_i(K')|^2 = (n+1)\left(\frac{J_{n+1}(2\pi K)}{\pi K}\right)^2 \cdot F(m\theta); \tag{6}$$

In this

$$F(m\theta) = \begin{cases} 2\cos^2(m\theta),\ i \text{ is even no.} \\ 2\sin^2(m\theta),\ i \text{ is odd no.} \end{cases} \Bigg\}\, m \neq 0$$
$$F(m\theta) = 1, \quad m = 0;$$

$J_{n+1}(\quad)$ is a Type I Bessel function. n and m are respectively the radial order number and the angular order number associated with Zernike polynomial forms. θ is the angular component. One takes equation (6) and substitutes into equation (5). In conjunction with that, one assumes that turbulent flows are uniform. After going through simplification, one obtains

$$C_{ii}(\lambda_1, \lambda_2) = \left(\frac{D^2}{r_{01} r_{02}}\right)^{5/6} \cdot P_i(\lambda_1, \lambda_2); \tag{7}$$

$$P_i(\lambda_1, \lambda_2) = 1.95(n+1)\int_0^\infty dk k^{-14/3} J_{n+1}^2(k) \left[\frac{\sin(Ak^2)}{A} + \frac{\sin(Bk^2)}{B}\right]; \tag{8}$$

In this, D = 2R. r_{01} and r_{02} are respectively atmospheric coherence lengths corresponding to λ_1 and λ_2 .

$$A = \frac{2L(k_1+k_2)}{k_1 k_2 D^2},\ B = \frac{2L(k_1-k_2)}{k_1 k_2 D^2}.$$

When $k_1 = k_2$, equation (7) and equation (8) change to be

$$C_{ii}(\lambda_n, \lambda_n)=\left(\frac{D}{r_{0n}}\right)^{5/3}P_i(\lambda_n, \lambda_n); \tag{9}$$

$$P_i(\lambda_i, \lambda_j)=1.95(n+1)\int_0^{\infty}dk\, k^{-20/3}J^2_{n+1}(k)\left[\frac{\sin(ck^2)}{c}+k^2\right]; \tag{10}$$

In this, $c = 4L/k_jD^2$. Here $C_{ii}(\lambda_i, \lambda_j)$ 219
is different from the results in reference (4). This is because we considered the effects of diffraction, and reference (4) is only the result of a geometrical optic approximation. The influence of diffraction effects depend on the magnitude of $4L/K_jD^2$, when $4L/K_jD^2$ is very small, the influence of diffraction effects is not great. However, the greater $4L/k_jD^2$ is, the greater the influence of diffraction effects also is. In Fig.1 we give partial calculation results for $C_{ii}(\lambda, \lambda)$ (taking geometrical optic approximation values and normalizing or unitizing them). In this, the curves respectively correspond to n = 3,5,7,10. It is possible to see that long wave diffraction effects are relatively larger. In particular, the correlations for higher order quantity coefficients receive even greater influence from diffraction effects. Besides this, the calculation results for $P_i(\lambda_1, \lambda_2)$ clearly show that, for different wavelengths, although $P_i(\lambda_1, \lambda_2)$ for lower order quantities are basically the same, $P_i(\lambda_1, \lambda_2)$ for higher order quantities, however, will show definite differences. It is precisely the existence of this type of difference which makes taking phase data for one type of wavelength unable to perfectly correct the phase distortions of a different type of wavelength.

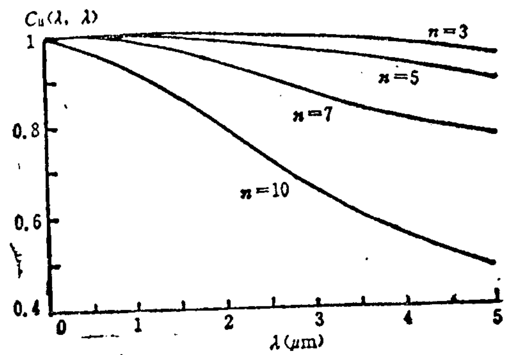


Fig. 1 Normalized correlation function $C_{ii}(\lambda, \lambda)$ VS. wavelenyth λ

III. RESIDUAL PHASE ERROR

From the correlation functions given in the section above, it is possible to obtain residual phase errors produced by atmospheric dispersion. Assuming that we take φ_2 phase information for beacon light with wavelength λ_2 in order to rectify the phase distortion φ_1, for transmitted light beams with wavelength λ_1, then the residual phase error after rectification is

$$\varphi_c = \varphi_1 - \varphi_2, \tag{11}$$

The mean square error is

$$\Delta_c^2 = \iint d^2\rho \langle (\varphi_c(\rho))^2 \rangle, \tag{12}$$

Making use of equation (3) and equation (11) as well as on the basis of the orthogonality of Zernike polynomial forms, equation (12) is capable of being expressed as

$$\Delta_c^2=\sum_i[C_{ii}(\lambda_1,\ \lambda_1)+C_{ii}(\lambda_2,\ \lambda_2)-2C_{ii}(\lambda_1,\ \lambda_2)] \tag{13}$$

C_{ii} is given by equations (7) and (9). Because of this

$$\Delta_c^2=\left(\frac{D}{r_{01}}\right)^{5/3}\sum_i\left[P_i(\lambda_1,\ \lambda_1)+\left(\frac{\lambda_1}{\lambda_2}\right)^2P_i(\lambda_2,\ \lambda_2)-2\left(\frac{\lambda_1}{\lambda_2}\right)P_i(\lambda_1,\ \lambda_2)\right] \tag{14}$$

Under the conditions of L = 10 km, D = 4 m, $\lambda_1=1\,\mu m$ the calculation results for Δ_c^2 are given in Fig.2.

Obviously, it is only when beacon wavelength λ_2 and transmitted wavelength λ_1 are identical that the residual phase error is zero. When $\lambda_1/\lambda_2<1$ that is, using long wavelengths to rectify short wavelengths, due to increases and reductions in phase distortion following along with wavelengths, the results will compensate inadequately. Moreover, residual phase error follows reductions in λ_1/λ_2 with increases, finally tending toward a value which is not compensated for. When $\lambda_1/\lambda_2>1$ 时, that is, using short wavelengths to rectify long wavelengths, the amount of rectification or correction is larger than the actual amount of distoriton. Residual phase error follows increases in λ_1/λ_2 and rapidly increases. When λ_1 exceeds λ_2 by a certain value, residual phase error reaches a value which has no compensation. If one uses even shorter beacon wavelengths, one inevitably will make compensated results even rougher than those when there is no compensation. The above analysis clearly shows that, with these types of correction or rectification methods, it is only when one uses beacon light waves which are very close to transmitted wavelengths that one will have relatively small residual phase errors, therefore obtaining relatively good compensation results.

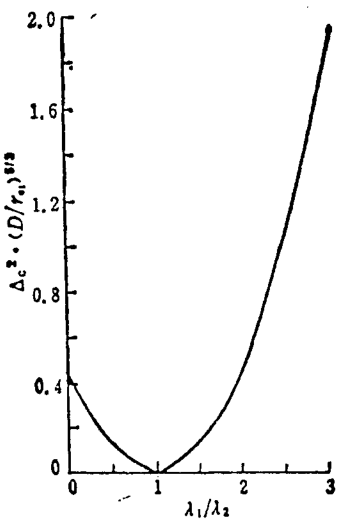
In order to obtain even better compensation results, it is possible to take $\lambda_2/\lambda_1\ \varphi_2$ and have it be the phase correction amount in order to compensate for a transmitted light beam phase distortion φ_1. At this time, the residual phase square difference is

$$\Delta_o'^2=\sum_i[\sigma_{ii}(\lambda_1,\lambda_1)+(\lambda_2/\lambda_1)^2\cdot\sigma_{ii}(\lambda_2,\lambda_2)-2\lambda_2/\lambda_1\sigma_{ii}(\lambda_1,\lambda_1)], \tag{15}$$

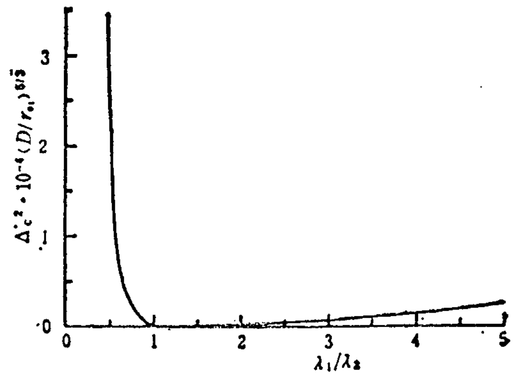
After going through rearrangements, the equation above becomes

$$\Delta_o'^2=\left(\frac{D}{r_{01}}\right)^2[P_i(\lambda_1,\lambda_1)+P_i(\lambda_2,\lambda_2)-2P_i(\lambda_1,\lambda_2)], \tag{16}$$

We did calculations for $\Delta_o'^2$ in different situations. The results clearly showed that residual phase errors were very greatly reduced. Fig.3 is the calculation results for L = 10 km, D = 4 m, $\lambda_1=1\,\mu m$. In a comparison between Fig.3 and Fig.2, it can be clearly seen that $\Delta_o'^2$ is much smaller than Δ_o^2. This is particularly true when λ_1/λ_2 is $>$ 1, and $\Delta_o'^2$ is very small. For example, when $\lambda_1/\lambda_2=3$, $\Delta_o'^2$ is $10^{-5}(D/r_{01})^{5/3}$. This is reduced more than 5 orders of magnitude. When λ_1/λ_2 is $<$ 1, $\Delta_o'^2$ is slightly larger. However, straight through to when $\lambda_1/\lambda_2=0.5$时, $\Delta_o'^2$ is still only $2.5\times10^{-4}(D/r_{01})^{5/3}$. There is still a 3 order of magnitude reduction.



(λ_1=1 μm, L=10 km, D=4 m)
Fig. 2 Residual Phase error Δ_0^2



(λ_1=1 μm, L=10 km, D=4 m)
Fig. 3 Residual phase error $\Delta_0^{2\prime}$

IV. BRIEF SUMMARY

The results of the analyses above clearly show that, due to turbulent flow effects and wavelength being related, using a type of wavelength to probe or survey phase information is only capable of carrying out perfect rectification or correction on phase distortions of the same wavelength. When one is using dual wavelength adaptive optics systems to rectify or correct atmospheric turbulence effects, directly taking the phase distortion of beacon light λ_2 as the phase correction amount is only capable of partially correcting phase distortions associated with transmitted light λ_1 . When the differences between the two wavelengths are relatively large--in particular, when $\lambda_1 \gg \lambda_2$ --very large residual phase errors will be produced. In order to reduce residual phase errors, it is possible to take the product of beacon phase errors and the

ratio of wavelength values λ_2/λ_1 to act as the phase correction amount. In this way, in geometrical optic approximations, one should completely rectify or correct phase distortions in transmitted light beams. However, due to the existence of diffraction effects, this type of correction or rectification will also produce residual phase errors. Residual phase errors produced by diffraction effects, even if the differences between the two wavelengths are relatively large, are not generally great. Because of this, the requirements for beacon wavelengths are very greatly relaxed. During concrete applications, it is appropriate to select for use beacon light with very short wavelengths. Generally, it is possible to use visible light band beacons to correct or rectify phase distortions in close infrared or infrared waveband lightwaves..

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